FALL 2019: MATH 558 QUIZ 7

Each question is worth 2 points.

(i) Let R be a commutative ring. Define what it means for u to be a unit.

Solution. $u \in R$ is a unit if it has a multiplicative inverse, i.e., there exists $u^{-1} \in R$ such that $u \cdot u^{-1} = 1$.

(ii) True or false: Every non-zero element in a field is a unit.

Solution. True.

(iii) Give an example of a unit different from $\overline{1}$ in the ring $\mathbb{Z}_4$.

Solution. $\overline{3}$, since $\overline{3} \cdot \overline{3} = \overline{1}$.

(iv) Let R be an integral domain. Define what it means for p to be an irreducible element.

Solution. $p \in R$ is irreducible, if whenever $p = a \cdot b$, for $a, b \in R$, either a is a unit or b is a unit.

(v) Let R be an integral domain. Define what it means for R to be a Euclidean domain.

Solution. R is a Euclidean domain if there exists a function $v : R \backslash 0 \to \{0, 1, 2, \cdots\}$ satisfying:

(i) For every pair of non-zero elements a, b in R, there exist $q, r \in R$ such that $b = a \cdot q + r$ and either $r = 0$ or $v(r) < v(a)$.

(ii) $v(a) \leq v(ab)$, for all nonzero $a, b \in R$.